Lectures on Electromagnetic theory I

PH 2151

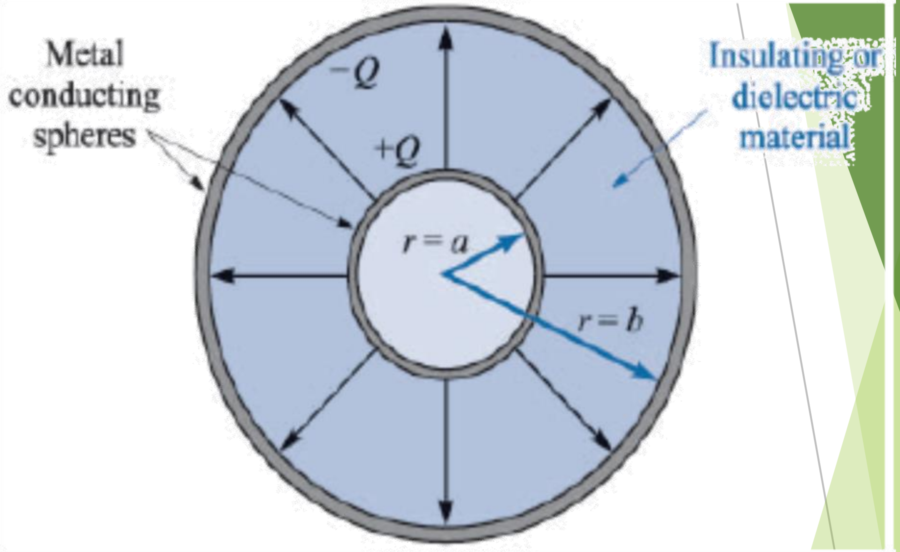
Lecture 4
(Electric flux density ,Gauss's law)

Prof. Salwa Saad Mohamed

Faraday's experiment and electric flux density

- Faraday concluded that there was some sort of electric flux **ψ** from inner sphere which has total positive charge **Q** to outer sphere.
- **Ψ =Q** in SI units .

- Electric flux density D is a vector field and measured in coulombs per square meter.
- The relation between **D** and **E** is $\mathbf{D}=\varepsilon_o \mathbf{E}$ in free space.



The electric flux in the region between a pair of charged concentric spheres. The direction and magnitude of D are not functions of the dielectric between the spheres

Gauss's law

The electric flux passing through any closed surface is equal to the total charge enclosed by that surface .

The charge enclosed might be:

- several point charges in which case Q=$\sum Q$n
- Or a line charge Q=$\int \rho_l \, dl$.
- Or a surface charge Q=$\int_s \rho_s$ ds.
- Or a volume charge Q= $\int_{vol} \rho_v \, dv$

Gauss's law

Examples :

- Find the electric flux crossing a surface S containing a line charge $0 \leq l \leq \pi$ m, with charge density

$$\rho_l = -5 \sin(l/2)\ c/m .$$

- Find the electric flux ψ crossing a surface S containing a disk with a radius 4 m and has charge density

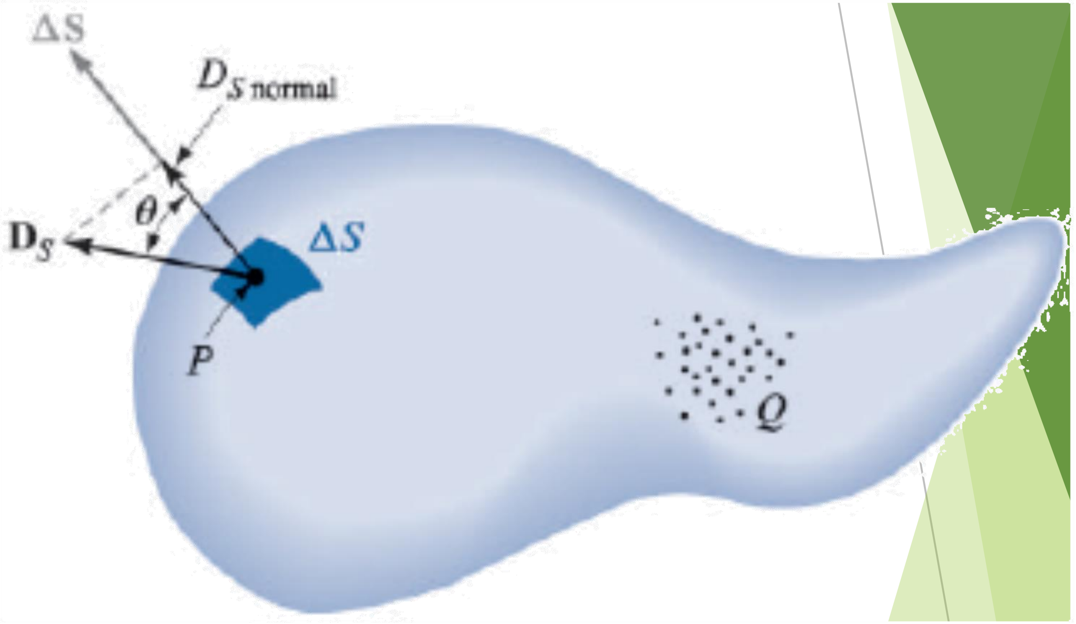
$$\rho_s = \frac{\sin^2\varphi}{2\rho}\ c/m^2$$

- Find the total charge Q on the volume $0 \leq x \leq 1$m , $0 \leq y \leq 1$m , $0 \leq z \leq 1$m, If the volume charge density

$$\rho_v = 30\, x^2 y\ \ \mu.C/m^3 .$$

The mathematical formulation of Gauss's law

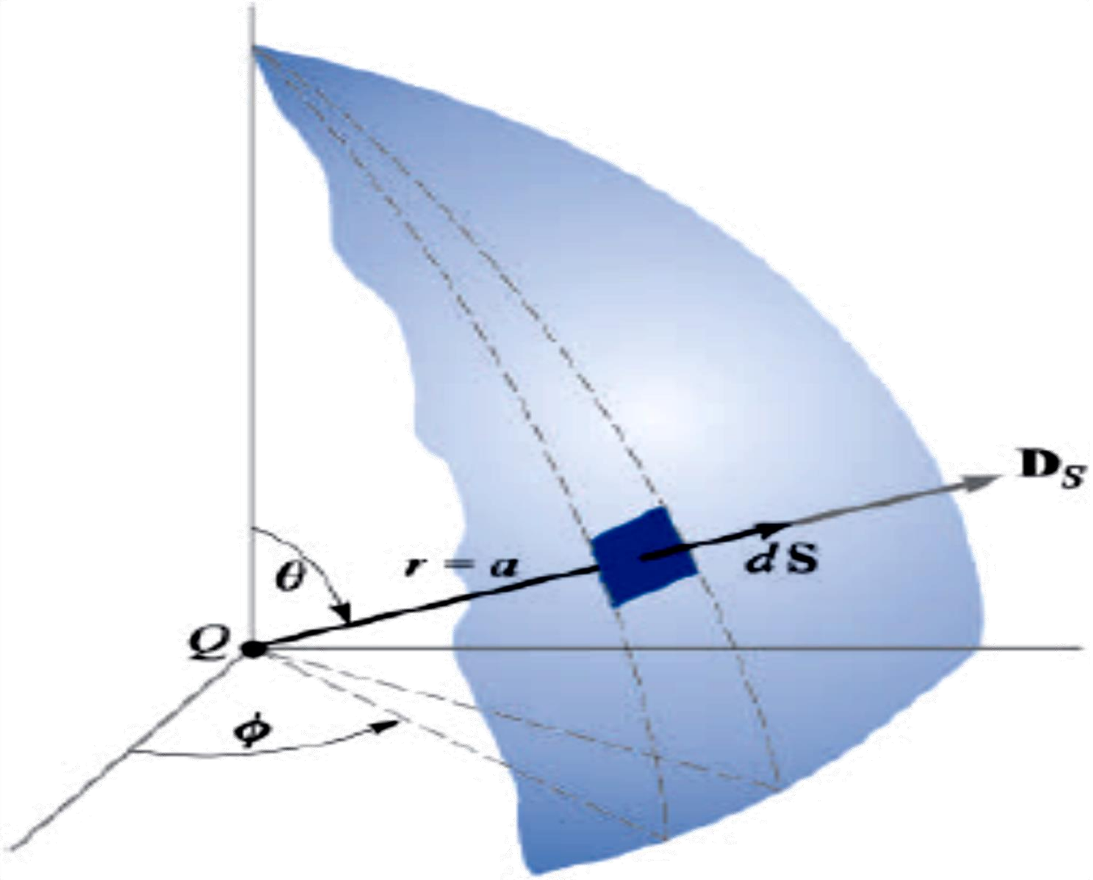
- $\Delta\psi$ =flux crossing ΔS = $D_{Snorm} . \Delta S = D_s \cos\theta \, \Delta S$ = $\boldsymbol{D_s} . \Delta S$
- $\Psi = \int d\psi = \oint D_s$ .ds
- The integrations is to be performed over a closed surface (gaussian surface). We have the mathematical formulation of Gauss's law



$$\Psi = \oint_S \mathbf{D}_S \cdot d\mathbf{S} = \text{charge enclosed} = Q$$

Faraday's experiment and Gauss's law

Application of Gauss's law to the field of a point charge Q on a spherical closed surface of radius a.
The electric flux density D is everywhere normal to the spherical surface and has a constant magnitude at every point on it.



Applications of Gauss's law Symmetrical charge distributions

- The conditions of Gaussian surface ?

1) **Point charge** : D = $\frac{Q}{4\pi r^2}\,\boldsymbol{a_r}$, E = $\frac{Q}{4\pi\varepsilon_0 r^2}\,\boldsymbol{a_r}$

The field falls off inversely with the square of the distance to the point charge

2) **Line charge** : D = $\frac{\rho_l}{2\pi\rho}\mathbf{a}_\rho$ E = $\frac{\rho_l}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho$

The field falls off inversely with the distance to the line charge

3) **A sheet of charge**: D = $\frac{\rho_s}{2}\,\boldsymbol{a_N}$ E = $\frac{\rho_s}{2\varepsilon_0}\,\boldsymbol{a_N}$

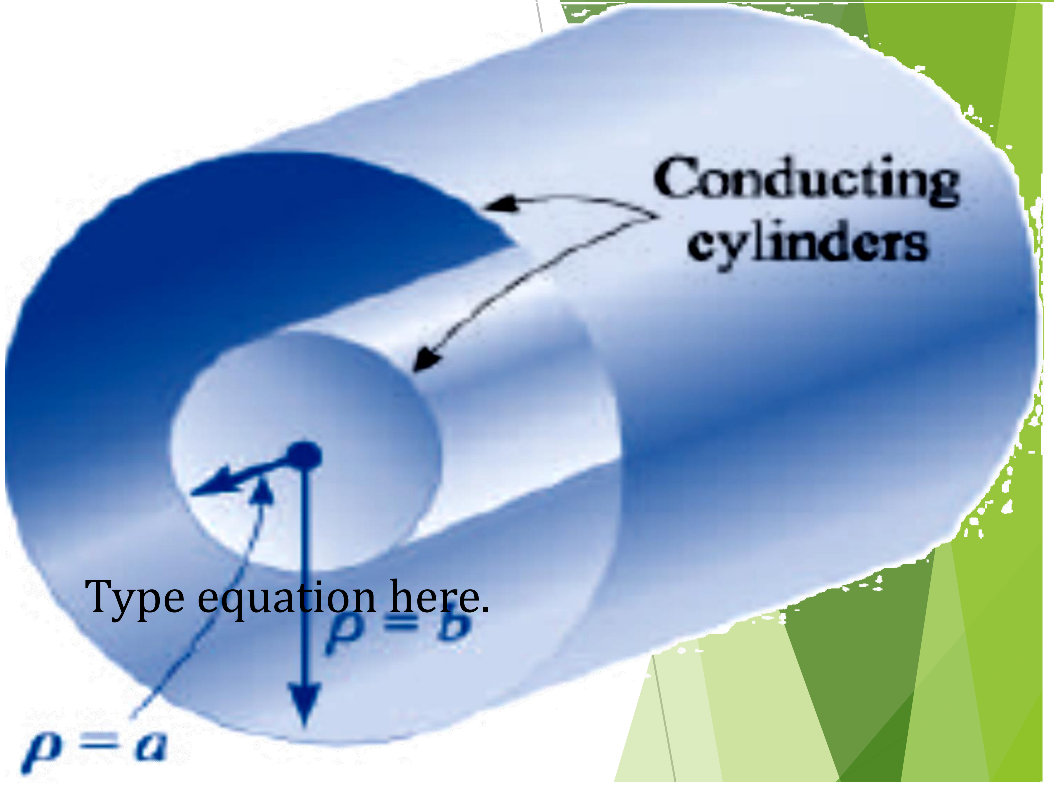
The field is constant in magnitude and normal to the sheet

The field between the parallel plates of air capacitor

E = $E_+ + E_- = \frac{\rho_s}{\varepsilon_0}$

The problem of coaxial cable

- $D = \frac{\rho_L}{2\pi\rho}\boldsymbol{a_\rho}$
  $= \frac{a\rho_S}{\rho}\boldsymbol{a_\rho} \qquad (\boldsymbol{a} < \boldsymbol{\rho} < \boldsymbol{b})$
- $E = \frac{\rho_L}{2\pi\varepsilon_0\rho}\boldsymbol{a_\rho}$
  $= \frac{a\rho_S}{\varepsilon_0\rho}\boldsymbol{a_\rho} \qquad (\boldsymbol{a} < \boldsymbol{\rho} < \boldsymbol{b})$
- $D=E=zero$ at $\rho > b,\ \rho < a$



The two coaxial cylindrical conductors forming a coaxial cable provide an electric flux density within the cylinders

Problems

- 1)Given a 5 μc point charge at the origin of the spherical coordinate system find the total electric flux passing through $0 \le \theta \le \pi/2$
- 2)Given a 60 μc point charge located at the origin. Find ψ passing through:
  a) that portion of sphere r=26 cm bounded by $0<\theta<\pi/2$ and $0<\Phi<\pi/2$
  b) closed surface $\rho = 26$ cm and $z = \pm 26$ cm
- 3) Find the electric flux density D in the region about a uniform line of 8 nc/m lying along z axis in free space at $\rho = 3$ m .
- 4)Let us select 50 cm of coaxial cable having inner radius of 1mm and an outer radius of 4 mm .The space between the conductors is assumed to be filled with air. The total charge on the inner conductor is 30nc .We wish to know the charge density on each conductor ,and E , D fields at $\rho <$ 1 mm , 1 mm $< \rho <$ 4 mm , $\rho >$ 4 mm .
- 5)Two sheets of charge having a uniform charge density $\rho_s = m/c^2$. One has + ve charge and the second has the same -ve charge , they were put on the x axis at $x = \pm 1$ find E at $-1 < x < 1$, $x < -1$, $x > 1$.